Spin–dependent twist–4 matrix elements from the instanton vacuum: Flavor–singlet and nonsinglet

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Abstract

We estimate the twist–4 spin–1 nucleon matrix element f_2 in an instanton–based description of the QCD vacuum. In addition to the flavor–nonsinglet we compute also the flavor–singlet matrix element, which appears in next–to–leading order of the $1/N_c$–expansion. The corresponding twist–3 spin–2 matrix elements d_2 are suppressed in the packing fraction of the instanton medium, $\bar{\rho}/\bar{R} \ll 1$. We use our results to estimate the leading $1/Q^2$ power corrections to the first moment of the proton and neutron spin structure functions G_1, as well as the intrinsic charm contribution to the nucleon spin.

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1 Introduction

The structure of the nucleon as measured in deep–inelastic scattering is described by matrix elements of QCD operators of a certain twist (dimension minus spin). The moments of the structure functions at asymptotically large Q^2 are given by matrix elements of operators of twist 2, which have a simple interpretation as number densities of the quarks and antiquarks in the nucleon in the infinite–momentum frame. Power $(1/Q^2-)$ corrections to the asymptotic result are determined by matrix elements of operators of twist 3 and 4. The latter describe either the correlation of the quark fields with the non-perturbative gluon field in the target, or quark–quark correlations. In the polarized case the twist–3 and 4 matrix elements of lowest spin are that of the twist–4 spin–1 operator

$$\langle p, \lambda | \bar{\psi}_f \gamma_\alpha \widetilde{F}^{\beta\alpha} \psi_f | p, \lambda \rangle \;\; = \;\; 2M_N^2 \; f_{2,f} \; s^\beta, \tag{1}$$

and of the twist–3 spin–2 operator

$$\langle p, \lambda | \bar{\psi}_f \left(\gamma^\alpha \widetilde{F}^{\beta\gamma} + \gamma^\beta \widetilde{F}^{\alpha\gamma} \right) \psi_f | p, \lambda \rangle - \text{traces}$$
$$= \;\; 2 \, d_{2,f} \; \left[2p^\alpha p^\beta s^\gamma - p^\gamma p^\beta s^\alpha - p^\alpha p^\gamma s^\beta + (\alpha \leftrightarrow \beta) - \text{traces} \right]. \tag{2}$$

Here $\bar{\psi}_f, \psi_f$ denote the quark fields of flavor f,

$$\widetilde{F}^{\mu\nu} \;\; \equiv \;\; \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} \tag{3}$$

is the dual of the gauge field strength ($\epsilon^{0123} = 1$), p the nucleon four–momentum ($p^2 = M_N^2$), λ the helicity, and $s = s(\lambda)$ the polarization vector of the nucleon state, which satisfies

$$s \cdot p \;\; = \;\; 0, \qquad\qquad s^2 \;\; = \;\; -M_N^2. \tag{4}$$

The nucleon states are normalized according to $\langle p, \lambda | p', \lambda' \rangle = 2p^0 (2\pi)^3 \delta^{(3)}(\mathbf{p} - \mathbf{p}') \delta_{\lambda\lambda'}$. The matrix elements (1) and (2) can be taken either in a proton or neutron state.

The matrix elements (1) and (2) together determine the leading power corrections to the first moment of the spin structure function G_1 (the Bjorken and Ellis–Jaffe sum rules) [1, 2, 3]. The twist–3 matrix element d_2 also appears in the QCD expression for the third moment of the second spin structure function G_2, where it is not power–suppressed relative to the twist–2 part and can therefore be measured with good accuracy [4]. By studying these matrix elements in theoretical models of the nucleon one may hope to get some insight into the transition from the region of asymptotically large Q^2 ($Q^2 \gg 1 \, \text{GeV}^2$), where the Q^2–dependence of the structure functions is described by perturbative QCD, to the resonance region ($Q^2 \approx 1 \, \text{GeV}^2$) [5]. Eventually, this may help to understand also the discrepancy of the moments of the structure functions measured at $Q^2 \geq 1 \, \text{GeV}^2$ with the GDH sum rule for photoabsorption ($Q^2 = 0$) [6].

The twist–4 matrix element f_2, (1), also plays a role in the so-called "intrinsic" charm contribution to the nucleon spin [7]. Making use of an expansion in inverse powers of the charm quark mass, $1/m_c$, Franz *et al.* [8], correcting an earlier results [9, 10], related the

charm quark contribution to the axial current to the matrix element of the flavor–singlet twist–4 operator, (1). Model estimates of flavor–singlet f_2 can thus be used to estimate the intrinsic charm contribution to the nucleon spin. Knowledge of this contribution is a prerequisite for attempts to measure the gluon polarization through open charm production [11].

Here we report about an estimate of the spin–dependent twist–3 and 4 nucleon matrix elements, (1) and (2), within an instanton–based description of the QCD vacuum. In this approach the QCD ground state is approximately described as a "medium" of topological vacuum fluctuations — instantons and antiinstantons, with perturbative fluctuations about them [12, 13, 14]. This picture explains the dynamical breaking of chiral symmetry [15], which happens due to the fermionic zero modes associated with the individual (anti–) instantons, and manifests itself in the appearance of a dynamical mass of the quarks, accompanied by a coupling to Goldstone bosons — the pions. Building on that, this picture explains a host of phenomenological data on "vacuum structure" and hadronic correlation functions [14]. It also gives rise to a successful description of the nucleon as a chiral soliton, in the formal limit of a large number of colors, $N_c \to \infty$ [16, 17].

A crucial element in this approach is the fact that the instanton medium is dilute, *i.e.* the average size of the (anti–) instantons, $\bar{\rho} \approx (600\,\text{MeV})^{-1}$, is small compared to their average distance, $\bar{R} \approx (200\,\text{MeV})^{-1}$, with $\bar{\rho}/\bar{R} \approx 1/3$. The existence of this small parameter makes possible a systematic analysis of non-perturbative effects generated by the instantons. Matrix elements of quark–gluon operators of twist > 2 have been studied in this approach in Ref.[18]. In particular, it was shown there that the matrix element of the twist–3 operator, d_2, is suppressed relative to the twist–4 one, f_2, by a factor of $(\bar{\rho}/\bar{R})^4$, and thus

$$\underset{\text{twist-3}}{d_2} \quad \ll \quad \underset{\text{twist-4}}{f_2}. \tag{5}$$

In Ref.[18] the value of d_2 was estimated to be of the order of 10^{-3}; this prediction is confirmed by the recent results of the E155x experiment [4]. Thus, in the instanton vacuum the parametrically leading higher–twist effects are due to the twist–4 matrix element f_2. In this letter we present a quantitative estimate of f_2 in the instanton vacuum. In addition to the flavor–nonsinglet nucleon matrix element, which appears in leading order of the $1/N_c$ expansion and has been calculated in Ref.[18], we calculate here also the flavor–singlet one. This allows us to make predictions for the power corrections to both the proton and neutron spin structure functions G_1, as well as to estimate the intrinsic charm contribution to the nucleon spin. For a detailed description of the foundations of the approach employed in the present investigation here we refer to Refs.[18, 19].

2   Twist–4 matrix elements from the instanton vacuum

Chiral symmetry breaking by instantons. Following Diakonov and Petrov [15, 20], the

dynamical breaking of chiral symmetry in the instanton vacuum can be studied in the large–N_c limit, where it manifests itself in the appearance of a dynamical quark mass, $M \neq 0$, accompanied by a coupling of the quarks to a pion (Goldstone boson) field, $\pi^a(x)$, $(a = 1, \ldots N_f^2 - 1)$ which arises from the "bosonization" of the 't Hooft many–fermionic interaction induced by the instantons. The resulting low–energy dynamics is summarized by the effective Lagrangian

$$L_{\text{eff}} = \bar{\psi}(x)\left[i\partial\!\!\!/ - MF(\partial^2)\,U^{\gamma_5}(x)F(\partial^2)\right]\psi(x), \tag{6}$$

where

$$U^{\gamma_5}(x) = \frac{1 + \gamma_5}{2}U(x) + \frac{1 - \gamma_5}{2}U^\dagger(x), \qquad U(x) = e^{i\pi^a(x)\tau^a}, \tag{7}$$

is a unitary matrix variable containing the pion field, and $F(p^2)$ are form factors, proportional to the Fourier transform of the zero mode of the (anti–) instantons, which drop to zero for spacelike momenta of the order of $-p^2 \sim \bar{\rho}^{-2}$. It is crucial that the dynamical quark mass, M, is parametrically small compared to the ultraviolet cutoff, $\bar{\rho}^{-1}$:

$$M\bar{\rho} \sim \left(\frac{\bar{\rho}}{\bar{R}}\right)^2. \tag{8}$$

In particular, this implies that for spacelike quark momenta of the order $M \ll \bar{\rho}^{-1} \approx 600\,\text{MeV}$ one may neglect the form factors and consider the quark–pion coupling as effectively pointlike.

Nucleon as chiral soliton. The effective Lagrangian (6) serves as a basis for the calculation of correlation functions in the instanton vacuum within the $1/N_c$–expansion [15, 20]. In particular, it gives rise to a picture of the nucleon as a chiral soliton [16]. Baryon correlation functions in the large–N_c limit are characterized by a classical pion field, which in the baryon rest frame is of "hedgehog" form

$$U_c(\mathbf{x}) = e^{in^a\tau^a P(r)}, \qquad r \equiv |\mathbf{x}|, \qquad \mathbf{n} = \frac{\mathbf{x}}{r}. \tag{9}$$

Baryon states of definite spin/isospin and momentum quantum numbers emerge from quantizing the collective rotations and translations of the classical soliton

$$U(\mathbf{x}, t) = R(t)U_c(\mathbf{x} - \mathbf{X}(t))R^\dagger(t). \tag{10}$$

Since the soliton moment of inertia and mass are of order N_c, the angular velocity of the collective rotation,

$$\Omega = -iR^\dagger\dot{R}, \tag{11}$$

as well as the linear velocity of the translation, are of order $1/N_c$, the quantization of the collective motion can be performed within a $1/N_c$ expansion [16]. The Ω^2–contribution to the baryon mass gives rise to the N–Δ mass splitting, which is of order $1/N_c$. This approach allows also to compute matrix elements of operators between baryon states. Technically, this is done by representing the matrix element as a functional integral over
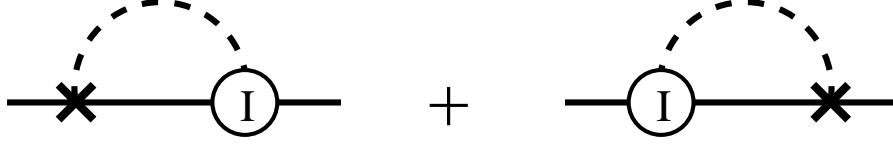
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Figure 1: Schematic illustration of the contributions to the matrix element of the quark–gluon operator, (1), in the instanton vacuum. The cross denotes the quark–gluon operator, the solid line the quark propagator, the dashed line the gauge field of the (anti–) instanton, and the circle the chirality–flipping 't Hooft interaction of the instanton with the quark mediated by the zero modes. The picture shows the situation for one light quark flavor; for N_f flavors the 't Hooft interaction is a $2N_f$–fermionic vertex, which at large N_c is equivalent to a coupling of the quark to the pion field ("bosonization"). Using the equations of motion of the effective low–energy theory, the instanton–induced vertex shown here can be converted to the chirally even effective operator (12). For details, see Refs.[18, 21].

the collective translations/rotations, which is then evaluated by expanding the integrand in Ω. For details we refer to Refs.[16, 17].

Twist–4 quark–gluon operator in the instanton vacuum. The instanton vacuum, being a microscopic model of the non-perturbative fluctuations of the gluon field, allows to estimate hadronic matrix elements of quark–gluon operators, such as the twist–4 operator of Eq.(1) [19, 18]. It is understood that the QCD operators are normalized at the scale defined by the inverse average instanton size, $\mu = \bar{\rho}^{-1}$. The main idea of this approach is that because of the diluteness of the instanton medium the gauge field operator in (1) can effectively be replaced by the field of a *single* (anti–) instanton, which then interacts with the quark fields through its zero modes (this is schematically illustrated in Fig. 1). As a result, the original QCD quark–gluon operator is replaced by a chirality–flipping "effective operator", which is to be evaluated in the effective theory where chiral symmetry is spontaneously broken. As has been discussed in Refs.[18, 21], this effective operator can be brought to a chirality–conserving form by making use of the equations of motion of the effective theory. For the QCD operator of interest, $\bar{\psi}_f \gamma_\alpha \widetilde{F}_{\beta\alpha} \psi_f$, the resulting operator is of the form [18]

$$- f_2^{\text{quark}} \, \bar{\psi}_f \gamma_\beta \gamma_5 \partial^2 \psi_f. \tag{12}$$

It is understood here that the momenta of the quark fields are of the order $p \sim M \ll \bar{\rho}^{-1}$. Here f_2^{quark} denotes the matrix element of the original chirality–flipping effective operator in a quark state, given by

$$f_2^{\text{quark}} \;\; = \;\; I_1(-p^2)|_{p^2=M^2}, \tag{13}$$

where $I_1(-p^2)$ denotes the quark loop integral defined by ($\bar{p}$ and $\bar{k}$ are Euclidean momenta, with $\bar{p}^2 = -p^2$)

$$I_1(\bar{p}^2) \;\; = \;\; \bar{\rho}^2 \int \frac{d^4\bar{k}}{(2\pi)^4} \frac{F(\bar{p}-\bar{k})\mathcal{G}(\bar{k})}{(\bar{p}-\bar{k})^2} \left[-\frac{3}{4}\frac{|\bar{k}|}{|\bar{p}|}C_1^{(1)}(\cos\theta) + \frac{1}{2}C_2^{(1)}(\cos\theta) \right], \tag{14}$$

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in which $C_n^{(1)}$ are the Gegenbauer polynomials (4–dimensional spherical harmonics) of argument $\cos\theta = (\bar{k} \cdot \bar{p})/(|\bar{k}||\bar{p}|)$, and the function $\mathcal{G}(\bar{k})$ describes the Fourier transform of the (dual) field strength of a single (anti–) instanton in singular gauge (see Ref.[18] for details):

$$\mathcal{G}(\bar{k}) = 32\pi^2 \left[\left(\frac{1}{2} + \frac{4}{t^2} \right) K_0(t) + \left(\frac{2}{t} + \frac{8}{t^3} \right) K_1(t) - \frac{8}{t^4} \right], \qquad t = |\bar{k}|\rho, \qquad (15)$$

with $K_n(t)$ the modified Bessel functions of the second kind. The numerical value of f_2^{quark} in the limit $M\bar\rho \to 0$ is [18]

$$f_2^{\mathrm{quark}} = \frac{3}{5} = 0.6. \qquad (16)$$

The instanton–induced effective operator (12) has the form of the axial current operator, but with an additional contracted derivative acting on the quark fields. Note that the coefficient, f_2^{quark}, is of order unity in the packing fraction of the instanton medium, $\bar\rho/\bar{R}$. Technically speaking, this happens because the quark loop integral I_1, (14), contains a would–be quadratic divergence, which is regularized by the instanton form factors and cancels the overall factor $\bar\rho^2$ in front of the integral. This implies that the hadronic matrix elements of this operator will also be parametrically large in the instanton packing fraction. We remark that for the twist–3 operator in the matrix element, d_2, (2), the effective operator would be proportional to $(M\bar\rho)^2 \sim (\bar\rho/\bar{R})^4$ and thus parametrically suppressed.

Nucleon matrix elements: $1/N_c$ expansion. The nucleon matrix elements of the effective operator, (12), can be computed using the same techniques as have been used in the calculation of nucleon matrix elements of vector and axial vector current operators (for a review, see [17]). In particular, we shall exploit the close relation of the effective operator, (12), with the axial current operator. It is known that within the chiral soliton picture of the nucleon the isovector and isoscalar component of the nucleon axial coupling constant appear in different orders in the expansion in powers of the angular velocity of the collective rotation of the soliton, $\Omega \sim 1/N_c$, (11). We therefore need to consider separately the isovector and isoscalar components of the twist–4 nucleon matrix element f_2, (1),

$$f_2^{(3)} = f_{2,u} - f_{2,d}, \qquad\qquad f_2^{(0)} = f_{2,u} + f_{2,d} \qquad (17)$$

(the matrix elements on the R.H.S. refer to the proton). We restrict ourselves for the moment to the $SU(2)$ flavor group (u, d), the extension to $SU(3)$ will be discussed below. For the isospin components defined by (17) the N_c–counting can immediately be inferred from that of the corresponding axial coupling constants of the nucleon:

$$M_N^2 f_2^{(3)} \sim N_c, \qquad\qquad M_N^2 f_2^{(0)} \sim 1. \qquad (18)$$

The isovector component of the spin–dependent twist–4 matrix element is leading in the $1/N_c$ expansion.

Isovector matrix element $f_2^{(3)}$. We now make a quantitative estimate of the isovector twist–4 nucleon matrix element, $f_2^{(3)}$. This can be done by considering the hypothetical

limit of large soliton size, in which one can perform an expansion of the matrix element in gradients of the classical pion field, (9). The result for $f_2^{(3)}$ in this approximation can immediately be written down by analogy with the well–known result for the isovector axial coupling, $g_A^{(3)}$ [16],

$$g_A^{(3)} \;\; = \;\; \frac{4N_c M^2 J}{9} \int d^3x \; \mathrm{tr}\, \left[-i\tau^a U_c^\dagger(\mathbf{x}) \partial_a U_c(\mathbf{x}) \right], \tag{19}$$

where J denotes the (Euclidean) quark loop integral

$$J \;\; = \;\; \int \frac{d^4\bar{k}}{(2\pi)^4} \, \frac{F^2(\bar{k}) \left[F^2(\bar{k}) - \bar{k}^2 F(\bar{k}) F'(\bar{k}) \right]}{\left[\bar{k}^2 + M^2 F^4(\bar{k}) \right]^2}. \tag{20}$$

(Note that the PCAC relation relates this integral to the pion decay constant, $4N_c M^2 J = F_\pi^2$.) The corresponding result for the isovector matrix element of the operator (12) is

$$M_N^2 f_2^{(3)} \;\; = \;\; \frac{4N_c M^2 f_2^{\mathrm{quark}} J_1}{9} \int d^3x \; \mathrm{tr}\, \left[-i\tau^a U_c^\dagger(\mathbf{x}) \partial_a U_c(\mathbf{x}) \right], \tag{21}$$

where

$$J_1 \;\; = \;\; \int \frac{d^4\bar{k}}{(2\pi)^4} \, \frac{\bar{k}^2 F^2(\bar{k}) \left[F^2(\bar{k}) - \bar{k}^2 F(\bar{k}) F'(\bar{k}) \right]}{\left[\bar{k}^2 + M^2 F^4(\bar{k}) \right]^2}, \tag{22}$$

which differs from J, (20), by an additional power of the quark virtuality, $\bar{k}^2$, in the integrand. For the ratio of $f_2^{(3)}$ to $g_A^{(3)}$ we thus obtain

$$\frac{M_N^2 f_2^{(3)}}{g_A^{(3)}} \;\; = \;\; -\frac{f_2^{\mathrm{quark}} J_1}{J}, \tag{23}$$

With the standard parameters of the instanton vacuum, $\bar{\rho} = (600\,\mathrm{MeV})^{-1}$ and $M = 350\,\mathrm{MeV}$, we find

$$\frac{f_2^{\mathrm{quark}} J_1}{J} \;\; = \;\; 0.49\,\bar{\rho}^{-2}. \tag{24}$$

With the experimental values $M_N = 940\,\mathrm{MeV}$ and $g_A^{I=1} = 1.25$, Eq.(23) thus gives[1]

$$f_2^{(3)} \;\; = \;\; -0.25\,. \tag{25}$$

The integral J_1 in the gradient expansion of $f_2^{(3)}$, (22), contains a would–be quadratic divergence, *i.e.*, it is parametrically of the order $J_1 \sim \bar{\rho}^{-2}$, contrary to the integral J in $g_A^{(3)}$, which is depends only logarithmically on $\bar{\rho}^{-1}$. Thus, in J_1 the dominant contribution comes from quark momenta of the order of the UV cutoff, $\bar{k} \sim \bar{\rho}^{-1}$. In principle

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[1]In the numerical estimate quoted in Ref.[18], $f_2^{(3)} = -0.11$, some contributions to the gradient expansion of the matrix element were missed. Hence the differences in the numerical values.

the representation of the instanton–induced effective operator by the local operator, (12), is accurate only for quark momenta of the order $\bar{k} \sim M \ll \bar{\rho}^{-1}$; for momenta of the order $\bar{\rho}^{-1}$ one should take into account the momentum–dependence (non-locality) of the instanton–induced effective operator. One should keep in mind that, anyway, the precise form of the UV cutoff of the effective low–energy theory depends on the details of the approximations made in the description of the instanton medium; only the gross features can be assumed to be generic. Thus, results for quantities given by "quadratically divergent" integrals, such as $f_2^{(3)}$, should generally be regarded as rough estimates ($\pm 50\%$). Whether or not to include the form factors in the effective operator is just part of this larger uncertainty. We have verified that using instead of (12) the "exact" non-local operator $-\bar{\psi}\gamma_\beta\gamma_5\,[I_1(\partial^2)/F(\partial^2)]\,\partial^2\psi$ changes the numerical result for $f_2^{(3)}$ by less than 10%, so using the local approximation to the operator seems completely justified.

Isoscalar matrix element $f_2^{(0)}$. The calculation of the $1/N_c$–suppressed isoscalar component of the twist–4 matrix element, $f_2^{(0)}$, is somewhat more involved, since it requires to take into account the time dependence of the saddle–point pion field, (10), to first order in the angular velocity of the soliton, Ω, (11). An important question is the degree of the would–be ultraviolet divergence of the isoscalar nucleon matrix element, $f_2^{(0)}$. This question can be investigated by gradient expansion. Expanding the average over quark fields of the flavor–singlet version of the operator (12),

$$(-i)f_2^{\text{quark}}N_c\,\text{Sp}\left[1_{\text{Flavor}}\;\gamma_\beta\gamma_5\partial^2\frac{1}{i\slashed{\partial}-MF(\partial^2)U^{\gamma_5}F(\partial^2)}\right],\tag{26}$$

in gradients of the (space– and time–dependent) background pion field, one finds that the functional trace is at most *logarithmically* divergent. This implies that the matrix element, which is obtained by substituting in (26) the slowly rotating pion field (10), and expanding to first order in the angular velocity, Ω, can also be at most logarithmically divergent. This circumstance is very fortunate, as it allows us to neglect the instanton–induced form factors in the quark–pion coupling in the Lagrangian, (6), and simply apply an external UV regularization in the form of a Pauli–Villars subtraction. (If the matrix element were quadratically divergent, as $f_2^{(3)}$ is, the result would have been strongly dependent on the specific form of ultraviolet cutoff applied.) With this simplification the calculation of the matrix element of the instanton–induced effective operator (12) becomes completely analogous to that of the isovector axial coupling constant, $g_A^{(0)}$ [22, 23], and can be performed using the same techniques.

A new feature in the calculation of the matrix element of the instanton–induced operator (12) compared to that of the axial current, $\bar{\psi}\gamma_\beta\gamma_5\psi$, is that with (12) contributions of order Ω^1 can appear from the action of the derivatives contained in the operator on the time–dependent isospin rotation matrices, $R(t)$. Such contributions were discussed in Ref.[24] in the context of the calculation of the isovector unpolarized quark distribution in the nucleon, and we refer to this article (in particular Section 3.1) for a detailed description of the Ω– ($1/N_c$–) expansion. After performing the Ω–expansion one expresses the remaining Green functions in the static background pion field, (9), in a basis of eigenfunctions of

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the quark single–particle Hamiltonian in the:

$$H = -i\gamma^0\gamma^i\nabla_i + \gamma^0 M U^{\gamma_5}, \qquad (27)$$
$$H|n\rangle = E_n|n\rangle. \qquad (28)$$

The result for the isoscalar matrix element then reads

$$f_2^{(0)} = \frac{N_c f_2^{\mathrm{quark}}}{M_N^2}\left\{ -\frac{1}{4I} \sum_{\substack{n \\ \mathrm{non-occup.}}} \sum_{\substack{m \\ \mathrm{occup.}}} \frac{1}{E_n - E_m} \right.$$

$$\times \left[\langle n|\sigma_3(E_m^2 - \mathbf{p}^2)|m\rangle\langle m|\tau_3|n\rangle + \langle n|\tau_3|m\rangle\langle m|\sigma_3(E_n^2 - \mathbf{p}^2)|n\rangle \right]$$

$$\left. -\frac{1}{2I} \sum_{\substack{n \\ \mathrm{occup.}}} E_n\langle n|\sigma_3\tau_3|n\rangle \right\}. \qquad (29)$$

Here I is the moment of inertia of the classical soliton, see Ref.[16]. The first term in the braces is a double sum over quark levels; it coincides with the expression for $g_A^{(0)}$ up to the insertions of the operator $E^2 - \mathbf{p}^2$ in the single–particle matrix elements, which come from the contracted derivative in the operator (12). The second term represents the abovementioned "new" contribution which results from the action of the derivative in the operator on the rotational matrices in the collective quantization. Up to the factor E_n inside the single sum over quark levels this contribution is identical to the expression for the *isovector* axial coupling constant, $g_A^{(3)}$.

In Eq.(29) the first term (the double sum over quark levels) turns out to be UV–finite and does not require regularization. The second term (the simple sum over levels, related to the isovector axial coupling constant), as it stands, contains a linear divergence when regularized *e.g.* with an energy cutoff. At the same time one observes an anomaly–type phenomenon in the sense that the sum over occupied quark levels is not equal to minus the sum over non-occupied ones, as it should be on grounds of the analyticity properties of the single–particle Green functions. However, both the linear divergence and the "anomaly" disappear when performing a Pauli–Villars (PV) subtraction, leaving behind a finite sum with usual behavior when summing over non-occupied instead of occupied states, in agreement with the above statement that the matrix element is at most logarithmically divergent. A similar phenomenon was encountered in the calculation of matrix elements of twist–2 operators of spin > 2 in the chiral quark–soliton model in Ref.[25]. In that case, too, the superficial power divergences and the "anomalies" simultaneously disappeared after PV subtraction. Note that the PV subtraction, contrary to regularization methods based on an energy cutoff, does not spoil the analyticity properties of the regularized sums.

The remaining PV regularized sums can be performed numerically, using the Kahana–Ripka method of diagonalizing the single–particle Hamiltonian in a basis of free quark states. The value of the PV cutoff is determined by fitting the pion decay constant [25],

$M_{PV} = 557\,\text{MeV}$. For the numerical estimate we use the variational soliton profile of Refs.[16, 25]. We obtain a numerical value of the flavor–singlet twist–4 matrix element of

$$f_2^{(0)} = 0.01. \tag{30}$$

The flavor–singlet matrix element is more than an order of magnitude smaller than the flavor–nonsinglet one, (25), which reflects the fact that the former is given by a "logarithmically divergent", the latter by a "quadratically divergent" expression.

Comparison with results of other approaches. The instanton result for the isovector twist–4 matrix element, $f_2^{(3)}$, agrees both in sign and in magnitude with the QCD sum rule results of Refs.[26, 27]. However, we disagree with these authors on the sign of the isoscalar matrix element (for a critical discussion of the QCD sum rule calculations, see Refs.[28]). We also disagree in the sign of the isovector matrix element with the bag model estimate of Ref.[2]; however, this model can hardly claim to give a realistic description of the quark–gluon correlations giving rise to the twist–4 matrix elements; *e.g,* it does not respect the QCD equations of motion.

Generalization to the $SU(3)$ flavor group. So far we have calculated the twist–4 matrix elements assuming the $SU(2)$ flavor group (u, d–quarks only). The results can easily be generalized to the $SU(3)$ case (including also s quarks). Here we consider the simplest scenario of perfect $SU(3)$ symmetry, *i.e.*, zero strange quark mass. In the chiral quark–soliton model the leading–N_c calculation gives a simple relation between the $SU(3)$ triplet and octet couplings, as the differences between them appear only due to the different states of collective rotations of the soliton. The ratio of the triplet and the octet part of the nucelon matrix element f_2 is therefore given by the ratio of the matrix elements of the respective Wigner D–functions in the rotational state corresponding to the nucleon spin, isospin and the hypercharge, $|\text{rot}\rangle = |S = T = 1/2, S_3 = 1/2, T_3 = 1/2\rangle$ (for details see [17]):

$$\frac{f_2^{(8)}}{f_2^{(3)}} = \frac{\langle\,\text{rot}\,|\,D_{83}^8\,|\,\text{rot}\,\rangle}{\langle\,\text{rot}|\,D_{33}^0\,|\,\text{rot}\rangle} = \frac{3}{7}. \tag{31}$$

From these relations we obtain the following numerical results for the twist–4 matrix elements in the $SU(3)$ symmetric case:

$$f_2^{(0)}|_{SU(3)} = 0.01 \tag{32}$$
$$f_2^{(3)}|_{SU(3)} = -0.25 \tag{33}$$
$$f_2^{(8)}|_{SU(3)} = -0.11. \tag{34}$$

3 Power corrections to proton and neutron spin structure functions

With the results obtained in the previous section we can now estimate the $1/Q^2$–corrections to the first moments of the proton and neutron spin structure functions, $G_1^{\text{p,n}}$. The expres-

sions obtained from the operator product expansion of QCD at tree level to order $1/Q^2$ are [1] [2]

$$\int_0^1 dx\, G_1^{p(n)}(x, Q^2) = \frac{1}{2} a_0^{p(n)} + \frac{M_N^2}{9Q^2} \left(a_2^{p(n)} + 4 d_2^{p(n)} + 4 f_2^{p(n)} \right).$$ (35)

Inclusion of radiative corrections would lead to a logarithmic Q^2–dependence of the coefficients. Here $a_n^{(0)}$ denote the spin–dependent matrix elements of the twist–2 spin–$(n+1)$ axial vector operator; the proton and neutron matrix elements (which include the quark charges) are given in terms of the $SU(3)$ singlet, triplet and octet components as

$$a_0^{p(n)} = \pm \frac{1}{6} a^{(3)} + \frac{1}{18} a^{(8)} + \frac{2}{9} a^{(0)},$$ (36)

with $+(-)$ for proton (neutron). We quote the expressions for three light quark flavors (u, d, s). For $n = 0$ the a_n coincide with the axial coupling constants of the nucleon:

$$a_0^{(3)} = g_A^{(3)}, \qquad a_0^{(8)} = g_A^{(8)}, \qquad a_0^{(0)} = g_A^{(0)}.$$ (37)

Of the power corrections the term proportional to $a_2^{p(n)}$ are the target mass corrections, while the terms proportional to $d_2^{p(n)}$ and $f_2^{p(n)}$ represent the dynamical higher–twist corrections; the proton and neutron matrix elements are defined in analogy to Eq.(36).

We now evaluate the dynamical power corrections using the instanton vacuum results. Following the basic philosophy of the instanton vacuum we put the parametrically suppressed twist–3 spin–2 matrix elements to zero (these matrix elements were estimated in Ref.[18] to be of the order of $\sim 1\%$ of the twist–4 ones). For the twist–4 matrix elements $f_2^{p(n)}$ we use the results (32)–(34), which imply

$$f_2^p = -0.046, \qquad f_2^n = 0.038.$$ (38)

To evaluate the twist 2 contribution we use the GRSV 2000 parameterization [29] ("standard scenario"), which includes the radiative corrections to the coefficient functions in NLO.[3] The results are shown in Fig.2. One sees that the dynamical twist–4 contribution has a rather small effect on the structure functions down to Q^2 of the order of $1\,\mathrm{GeV}^2$, in particular in the neutron. Note that the quantitative details may change when $SU(3)$ symmetry breaking effects are included in the twist–4 contribution.

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[2]Note that there is a mistake in Ref.[1] concerning the coefficients in this formula, as was noted in [2]; see also Ref.[3].

[3]To evaluate the target mass corrections we use instead of the twist–2 matrix element $a_2^{p(n)}$ the third moment of the twist–2 part of $G_1^{p(n)}$ calculated from the GRSV 2000 parametrization; the difference between the two is irrelevant in the present context.
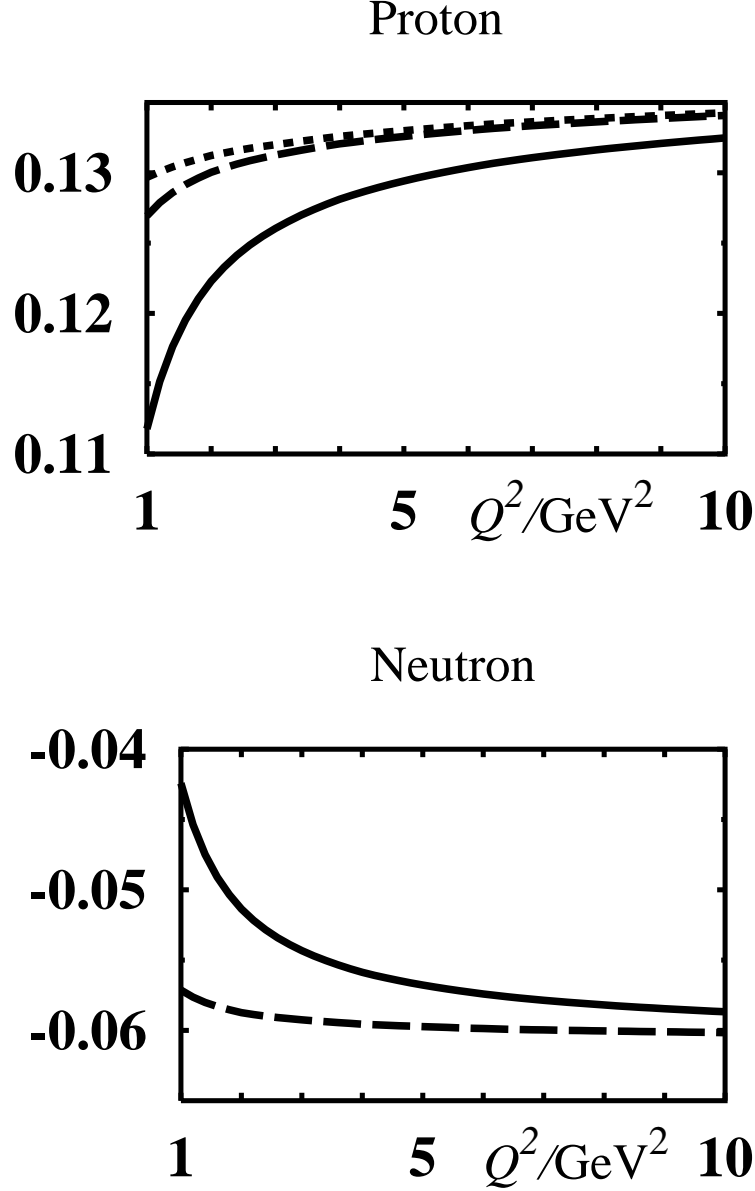
Figure 2: The Q^2–dependence of the first moment of the polarized structure function G_1 for the proton (top) and neutron (bottom). *Dashed lines:* Twist–2 contribution according to the GRSV 2000 NLO parameterization (standard scenario) [29]. *Dotted lines:* Sum of twist–2 contribution and target mass corrections. *Solid lines:* Total result, including also the twist–4 contribution due to the matrix elements f_2, as estimated in the instanton vacuum. (In the case of the neutron the target mass corrections to the first moment are very small, so we show only the pure twist–2 contribution.)

In Fig.3 we show the result for the first moment of the difference of proton and neutron structure functions, $G_1^p - G_1^n$ (the Bjorken sum). This quantity is of particular interest, since the Bjorken sum rule is a rigorous prediction of QCD, and the radiative corrections have been calculated to order α_s^3 [30]. We again use the GRSV2000 NLO parametrization to estimate the twist–2 part and target mass corrections. The power corrections now receive contributions only from the flavor–triplet twist–4 matrix element, $f_2^{(3)}$. Also shown in Fig.3 are the experimental results obtained in the analyses of the SMC [31], E143 [32], and E155 [33] experiments. One observes that the relatively large twist–4 correction obtained from the instanton vacuum improves the agreement of the theoretical prediction with the data; however, the present experimental errors are too large to allow for definite conclusions.
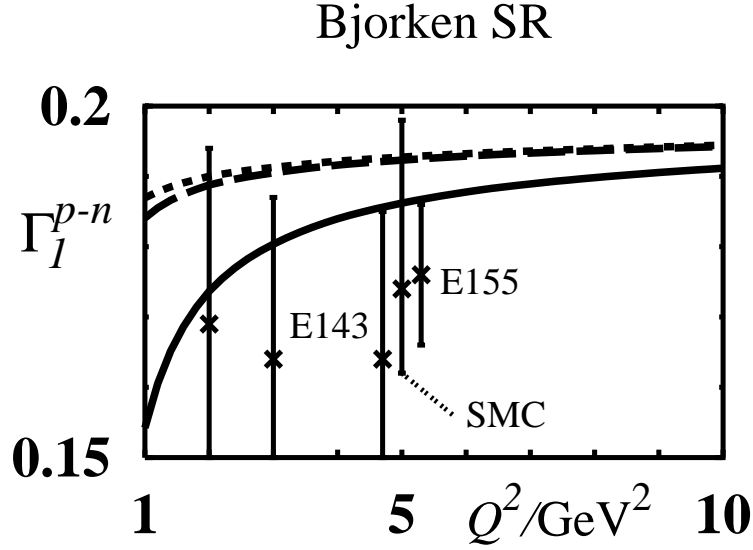

Figure 3: Same as Fig.2, but for the difference of proton and neutron structure functions (Bjorken sum rule). Also shown are the experimental results quoted by the E143 collaboration at $Q^2 = 2, 3$ and $5\,\mathrm{GeV}^2$ [32], and by the SMC [31] and E155 [33] collaborations at $Q^2 = 5\,\mathrm{GeV}^2$. (The abscissae of the data points at $Q^2 = 5\,\mathrm{GeV}^2$ have been shifted slightly in order to separate them in the plot.)

4   Intrinsic charm contribution to the proton spin

The isoscalar component of the twist–4 matrix element, $f_2^{(0)}$, (1), plays a role in the nucleon spin structure functions aside from power $(1/Q^2-)$ corrections, namely in determining the intrinsic charm contribution to the nucleon spin. Up to power corrections, the first moment

of the proton spin structure function for three light flavors (u, d, s) is given by [*cf.* Eqs.(35) and (36)]

$$\int_0^1 dx\, G_1^p(x, Q^2) \;=\; \frac{1}{12} g_A^{(3)} + \frac{1}{36} g_A^{(8)} + \frac{1}{9}\, g_A^{(0)} \;+\; O\!\left(\frac{1}{Q^2}\right). \tag{39}$$

When the charmed quark is included, its contribution can be expressed as a modification of the $SU(3)$ flavor–singlet axial coupling constant in (39):

$$g_A^{(0)} \;=\; g_{A,\text{light}}^{(0)} \;+\; 2g_A^{(c)}, \tag{40}$$

where $g_{A,\text{light}}^{(0)}$ contains the $SU(3)$–singlet contribution from the light flavors only, and $g_A^{(c)}$ is defined as

$$\langle p, s | \bar{c}\gamma_\beta\gamma_5 c | p, s \rangle \;=\; -2g_A^{(c)} s_\beta. \tag{41}$$

In the proton, where the charmed quarks occur in the form of virtual $\bar{c}c$ pairs, the charm quark axial current operator can be approximated by an operator involving only light quarks (u, d, s) and the gluon fields, using the heavy–quark mass $(1/m_c-)$ expansion to integrate out the charm degrees of freedom. Refs.[8, 7], correcting earlier results of Refs.[9, 10], quote the following result for the nucleon matrix element of the charm axial current to order $1/m_c^2$:

$$\langle p, s | \bar{c}\gamma_\beta\gamma_5 c | p, s \rangle \;=\; \frac{\alpha_s}{12\pi m_c^2} \, \langle p, s | \sum_{f=u,d,s} \bar{\psi}_f \gamma_\alpha \widetilde{F}_{\beta\alpha} \psi_f | p, s \rangle, \tag{42}$$

from which follows that

$$g_A^{(c)} \;=\; -\frac{\alpha_s}{12\pi} \frac{M_N^2}{m_c^2} f_2^{(0)}. \tag{43}$$

Using our result for the $SU(3)$ flavor–singlet matrix element, $f_2^{(0)}$, Eq.(32), we can now estimate the charm axial constant, $g_A^{(c)}$. With $\alpha_s(\mu = m_c) = 0.39$ and $m_c = 1.15\ldots 1.35\,\text{GeV}$, we find

$$g_A^{(c)} \simeq -1 \times 10^{-4}. \tag{44}$$

The charm quark contribution to the proton spin is very small, mostly because of the smallness of the flavor–singlet twist–4 matrix element, $f_2^{(0)}$.

5 Summary

The instanton vacuum with its inherent small parameter — the packing fraction of the instantons, $\bar{\rho}/\bar{R}$ — implies a parametrical (and numerical) hierarchy of the spin–dependent twist–3 and 4 matrix elements: twist–3 $\ll$ twist–4. The unusually small value for the twist–3 matrix element $d^{(2)} \sim 10^{-3}$ obtained from the instanton vacuum [18] agrees well with the precise measurements of G_2 in the E155x [4] experiment (and, also, the older E143 [32] and E155 [33] results). Note also that recently revised lattice calculations of d_2 [34], properly accounting for operator mixing effects, confirmed the small value predicted by the instanton vacuum.

In this paper we have presented numerical estimates of the twist–4 matrix elements f_2, including also its flavor–singlet component. Our approach predicts a big numerical difference between the flavor–nonsinglet and singlet matrix elements, $|f_2^{(3)}| \gg |f_2^{(0)}|$, in contrast to QCD sum rules, which tend to give values of the same order of magnitude. Phenomenologically, this would imply much larger power corrections to the Bjorken than to the Ellis–Jaffe sum rule, a prediction which can in principle be tested experimentally. We have also pointed out that the small value of $f_2^{(0)}$ implies a very small intrinsic charm contribution to the nucleon spin.

The approach laid out here can be extended in many ways; for example, one can compute also the matrix elements determining power corrections to higher moments of the structure functions and restore the the x–dependence of the twist–4 contribution. This would allow a more direct comparison of the higher–twist corrections with experimental data.

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